Sequential decisions in a bank run model*

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Abstract

We study a Diamond-Dybvig model with sequential move. If depositors observe each previous action (both withdrawals and waitings), then there are no bank runs in equilibrium. This result is a natural extension of the original Diamond-Dybvig model when depositors coordinate on running. On the contrary, if only withdrawals are observed, then runs appear in equilibrium. In the third setup we allow (but do not require) agents to report that they wait, and if the cost of reporting is moderate, then truth-telling will be the unique equilibrium and no report about waiting will be made in equilibrium. It suggests that by enriching the communication between the bank and the depositors bank runs resulting from coordination failure can be prevented.

1 Introduction

According to Gorton and Winton (2003) a bank run occurs "if the depositors of a single bank suddenly demand cash in exchange for their deposits". In the theoretical debate about runs, two well-distinguishable strands of literature emerged regarding the cause of this sudden demand. The first one refers to fundamental reasons, like worsening macroeconomic conditions, and asymmetric information about these fundamentals, see for example Gorton (1988) and Chari and Jagannathan (1988). The other view studies the possibility of self-fulfilling runs which may happen as a result of expecting other agents to withdraw, possibly without any fundamental reason. The seminal paper in this branch of the literature is Diamond and Dybvig (1983), which shows how risk-sharing concerns and private information about liquidity shocks provide a rationale for banks, which try to implement the first best allocation. This can be achieved through

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a simple demand-deposit contract, which allows for possible runs. It happens if all agents attempt to withdraw their money from the bank because they expect it to fail. Hence, even well-functioning banks may be subject to self-fulfilling runs. The authors provided policy solutions (suspension of convertibility and deposit insurance) which make the no-run equilibrium prevail costlessly for the case when the aggregate demand for withdrawals is constant. Diamond and Dybvig models the decision-making of the agents as a simultaneous-move game, which prevailed in the subsequent literature as well.

Our analysis takes a different stance, because we replace the simultaneous-move approach by sequential decision-making. The main objective of this paper is to study the consequences of sequential decisions in different informational setups. More concretely, we try to answer the question whether the ex-ante efficient allocation can be implemented in a sequential setup. The simultaneous-move approach has been criticised mainly on the ground of historical facts which show that agents take into account the opinions and actions of other agents, so decisions are not made simultaneously. It is enough to read descriptions of the banking panics in the nineteenth century (Sprague (1910)) or in the 1930´s (Friedman and Schwartz (1971), Wicker (2001)) which show that there were withdrawing waves, so people could observe other people's actions. Banking panic episodes during the Great Depression lasted for months and withdrawals did not start at once in all panic-stricken region. A common feature of these descriptions is that as people realize that many other people try to withdraw their money, they decide to follow suit. A more recent bank-run episode is studied by Starr and Yilmaz (2007) who deal with a banking panic which affected Turkey's Islamic financial houses in 2001. Based on detailed depositor information the authors carry out a VAR-analysis studying the behavior of depositors of different size (small, medium and large). In all of the groups, depositors were quite responsive to their peers and to the observable behavior of depositors of other groups showing that agents observed and reacted to others´ decisions. Iyer and Puri (2008) examine depositor level data for a bank that faced a run in India in 2001. They show, *inter alia*, that social network effects, that is, observing what agents - to whom one is connected - do are important regarding the decision-making. Experiments on bank run (Garratt and Keister (2008), Schotter and Yorulmazer (2008)) also support the use of a sequential setup.[1]

Our study can be seen as an alternative answer to Green and Lin´s (2000) question asking what is missing in Diamond and Dybvig´s model. They argued that the possibility of run in Diamond and Dybvig as a coordination problem and the pervasiveness of run in the subsequent literature is due to the simple contracts the bank offer. By introducing complex contracts which condition payments on the history of actions of previous agents but preserving the simultaneous-move approach, they show that the miscoordination disappears. Our way of answering the same question is to consider what happens in a sequential setup with simple contracts and we also obtain that in certain setups

[1] The conclusion of Garratt and Keister (2008) has that "... the standard approach of modelling bank runs using a one-shot, simultaneous-move game may not be the most appropriate one."

runs cannot emerge in equilibrium.

Sequentiality implies that the we have to specify the information subsequent agents observe. The source of the information in our model is the bank which shares all the available information it has with the current agent who is deciding. Following the literature, we assume the bank as the coalition of depositors (or equivalently as a programmed machine, see Wallace (1988)) which acts in interest of the depositors.[2] First, we take the approach by Green and Lin (2003) and impose a direct revelation mechanism. Each agent has to report her decision (to wait or to withdraw) to the bank. Our models differ in the following. In Green and Lin there is a huge withdrawal demand uncertainty, they allow the bank to write very sophisticated contracts[3], but restrict the agents´ information to have some notion about the position, but not knowing anything about the others´ decision. In contrast, we work with simple contracts and no demand uncertainty, but allow the bank to share available information with the agents. Our optimal contracts are simple ones which pay the same amount to any withdrawing agent. This trade-off in modeling choice does not change the conclusion that no bank run happens in equilibrium. As a next step, taking seriously the critique by Peck and Shell (2003) regarding the observability of waiting, we establish that if only withdrawals are observable, then run remains an equilibrium outcome. This is in line with Peck and Shell´s result who show it in a simultaneous-move model with aggregate uncertainty and complex contracts. In the last part, we try to bridge the gulf between the two previous results. In a novel way, we allow (but do not require) agents to inform the bank about their decision to wait and show that in this case run ceases to be an equilibrium and that in equilibrium patient agents will wait without reporting it. The possibility of reporting can be seen as the possibility of a richer communication between the bank and the depositors. This result is supported by findings of Iyer and Puri (2008) who analyzing a micro data set on a bank which has been run show that the longer and deeper the bank-depositor relationship is, the less likely are depositors to run.

Notice that our results only say whether bank run is or is not an equilibrium outcome due to pure coordination failure, but they remain silent about what happens when fundamentals worsen. The importance of studying this issue stems from the desire to design optimal institutions to avoid bank runs which cannot be explained by fundamental reasons (like the one in Iyer and Puri (2008)). These bank runs may set back considerably the financial intermediation and consequently the economic growth.

The existing vast literature on bank runs predominantly uses the simultaneous-move setup. To our best knowledge, there are two papers which use sequentiality as a modeling choice. The first is that by Zhu (2001), whose aim is to build a model which can explain the occurrence of fundamental runs without self-fulfilling prophecies. The model departs in many ways from Diamond and

[2] Hence, we assume away any possible agency problems. Such problems are addressed by Andolfatto and Nosal (2008).

[3] The optimal contracts in Green and Lin specify payments to withdrawing agents which depend in a complicated way on the history of reports to the bank.

Dybvig. Agents observe all previous actions which leads to a unique equilibrium, and there are no self-fulfilling runs. Runs occur if and only if agents perceive a low return on the asset, and the probability of bank run can be endogenously determined. Our paper is akin to Zhu´s in showing that if agents have to reveal their decisions and these are observable, then there is a unique equilibrium without run, but there are important differences. First, he obtains the result for a given first-period payment, while we do it for the optimal payment. Second, in the proof he uses subgame-perfection arguments although due to the multiplicity of possible alignments proper subgames are rare, so his arguments are theoretically questionable. The second paper is written by Chao Gu (2007) who models herding behavior in bank runs. She assumes away self-fulfilling runs[4] and focuses on fundamental-based runs which occur in the model solely due to the imperfect information on the productivity. Her main result is that given a demand-deposit contract, there exists a perfect Bayesian equilibrium in which the depositors withdraw if their expected utility are below the threshold, and wait otherwise. Hence, these papers focus on the equilibria resulting from a given information structure combined with signals about the fundamentals, without studying the possible coordination problems. On the contrary, we assume away the potential problems with the fundamentals and concentrate on the problems related to coordination and how the equilibrium changes in a sequential setup when the available information varies.

The remainder of the paper is organized as follows. Section 2 introduces the model. Section 3 discusses the equilibrium outcome when all previous actions are observable. Section 4 studies what happens if agents know only about previous withdrawals. In section 5, we discuss the consequences when agents are allowed but not obliged to report if they wait. Section 6 concludes.

2 The model

2.1 Environment and agents

There are three periods (T=0,1,2) and a single homogeneous good. Consider a finite number ($n > 2$) of agents. Each agent is endowed with 1 unit of the good in period 0. In period 0 each agent is identical, and faces a privately observed, uninsurable risk of being impatient (*imp*) or patient (*pat*). Thus, the type set is $\boldsymbol{\theta} = \{imp, pat\}$ and θ_i is agent $i's$ realized type. Nature chooses a constant number $p \in [2, n-1]$ which determines the number of the agents who are patient.[5] The rest of agents is impatient. The number of patient and impatient agents is common knowledge. Types are privately learnt in period 1. Impatient agents care only about consumption in period 1, whereas the patient agents value consumption in both periods. Hence, impatient agents always

[4] When a self-fulfilling run may arise, she assigns probability zero to this event.

[5] If everybody is of either type, then our problem becomes irrelevant. If there is only one patient agent, then the first-order conditions (to be derived later) imply that being truthful is a dominant strategy for her.

withdraw, whereas patient agents may withdraw or wait. Denote by (c_1, c_2) the consumption bundle of an agent in the two periods. We use the following utility function

$$u(c_1, c_2, \boldsymbol{\theta}_i) = u(c_1 + \boldsymbol{\theta}_i c_2),$$

where $\boldsymbol{\theta}_i$ is a binomial random variable with support $\{0, 1\}$. After realization of types, if $\theta_i = 0$, then the agent is impatient caring only about consumption in the period 1, otherwise she is patient. Assume that $u : R^2_{++} \to R$ is twice continuously differentiable, increasing, strictly concave, satisfies the Inada conditions and the relative risk-aversion coefficient $-cu''(c)/u'(c) > 1$ for every c. Agents are expected utility maximizers.

There is a constant-return-to-scale productive technology with the following returns:

T=0	T=1	T=2
-1	0	R
-1	1	0

with $R > 1$, so agents have to make a decision between $(0, R)$ and $(1, 0)$ in period 1. The long-term return, R, is constant.

2.2 The first best and the bank

If a planner could observe each depositor's type and assign an allocation based on these types, then the resulting first-best allocation would solve

$$\begin{aligned}&\max\ (n-p)u(c_1) + pu(c_2)\\ &s.t.\\ &(n-p)c_1 + [pc_2/R] = n\end{aligned}$$

In the formulation of the problem we imposed the optimality condition that the $n-p$ impatient agents consume only in period 1, whereas the patient agents consume only in period 2.

This problem yields the solution

$$u'(c_1^*) = Ru'(c_2^*),$$

which implies $R > c_2^* > c_1^* > 1$.

The rationale for a bank is the implementation of the first best. The bank pooles the resources and offers a simple demand deposit contract which specifies paying c_1^* to the withdrawing agents. The bank has to pay to withdrawing agents immediately c_1^* (unless it has run out of funds) and cannot make agents wait and condition payment on information which is not available at the time the agent wants to withdraw. A bank working this way is said to respect the sequential service constraint. Agents who have waited receive a pro rata share of the funds which were not withdrawn but were augmented by the productive technology. Formally, we define the period-2 consumption as

$$c_2(\mu) = \begin{cases} \max\left\{0, \frac{R(n-(n-\mu)c_1^*)}{\mu}\right\} \text{ if } \mu > 0 \\ 0 \text{ if } \mu = 0 \end{cases},$$

where μ is the number of agents who wait in period 1. As usual in the literature, depositors are isolated and no trade can occur among them in period 1.

Since the optimal payments in both periods are readily established by the parameters, obtaining the first best depends only on the actions of the patient agents, since impatient agents always withdraw in period 1. Hence, we focus on the decision of the patient agents, since the only source of a run is their possible miscoordination in the first period.

2.3 Decision, information and runs

The basic actions for any agent in the first period are withdrawal (wi) and waiting (wa). In one of the setups we will allow one more action, reporting a waiting (r). Throughout the paper we consider pure-strategy equilibria.

A main ingredient of the model is that we allow the bank to share the information it has with the agents if it helps to prevent runs. This is in line with the assumption that the bank maximizes the expected utility of the depositors. We view the bank as a programmed machine which given the parameters calculates c_1^* and then provides the agents with the available information and serves them if they withdraw, excluding the possibility that the bank gives misinformation.

The sequence of decision ($\theta^n = (\theta_1, ..., \theta_n)$) is exogenously determined in the following way. The number of patient agents (p) is known and nature chooses at random p agents in the line who will be patient. The remaining agents will be impatient. There are $\binom{n}{p}$ lines of length n with p patient agents, so these are the possible type vectors (or alignments). Each possible alignment has the same probability, $\frac{1}{\binom{n}{p}}$. Note that this assumption is the least informative possible, reflecting that we do not have a solid knowledge about the order in which agents go to the bank. Since our results do not depend on the distribution of alignments, this exogeneity assumption is not crucial. We suppose that neither the agents nor the bank know the alignment, they only know n and p.

Let $\theta_1^{i-1} \in \Theta_1^{i-1}$ denote the partial type vector up to agent $i-1$, and let $\theta_{i+1}^n \in \Theta_{i+1}^n$ stand for a feasible continuation type vector after agent i. Thus, $\theta_1^{i-1} = (\theta_1, ..., \theta_{i-1})$ and $\theta_{i+1}^n = (\theta_{i+1}, ..., \theta_n)$.

We assume that each agent decides only once. As a tie-breaking rule, we suppose that a patient agent who is indifferent between withdrawing and waiting will withdraw. This assumption is not crucial for the argument.

We define bank runs in a very broad sense. We interpret a patient agent withdrawing in the first period as a (partial) bank run.

We will work with three different information setup. First, we will impose a direct revelation mechanism, so that agents have to report their decision. Therefore, the bank has the exact history of the decisions, since both actions are observable. This mechanism is the same as in Green and Lin (2003), but in our setup agents get to know the history. It can be considered as the full information benchmark case. In the second setup we follow Peck and Shell (2003) who claim that it is more natural to think that only those agents contact the bank who want to withdraw, so only withdrawals are observable. In the last informational setup patient agents may report that they have waited and this report will be seen by later-coming agents. The reason of choosing these setups is that the first two relate closely to existing papers with a different modelling choices (simultaneous-move games with complex contracts) and lead to different conclusions regarding the possibility of bank runs. The third setup allows us to bridge the gap between the two results by showing that by changing slightly the game in the second setup we obtain the same conclusion as in the full-information case.

3 All previous decisions are observable

In this setup we require that agents state their action to the bank, so waitings become observable. The bank shares all the available information it has with the agents, so each agent knows the exact history of actions. This direct revelation mechanism has been applied by Green and Lin (2003) in an environment with complex contracts and simultaneous decision-making, where agents have some notion about their position in the line. We use a simple contract and agents know exactly what happened before in a sequential decision-making setup.

3.1 An example

To get the intuition of what happens consider the following example with four agents.[6] If there is only one patient agent, then there is no coordination problem. Two patient agents also coordinate easily, because since the best response to a waiting is to wait, so the first patient agent in the line will wait to induce the other one to do the same. A patient agent knows that she is the first patient agent in the line if previous agents have withdrawn. The interesting case is that with three patient agents and when all have to wait to make waiting worthwhile. Suppose that $u(c_2(\mu = 3)) > u(c_1^*) > u(c_2(\mu \leq 2)$ and $3c_1^* < 4$, so patient agents at position 1,2 and 3 would only want to wait if all the other patient agents wait. The optimal decision for a patient agent in the last position is easily defined. When she observes a history with 2 withdrawals she withdraws, otherwise she waits.

Any history containing two waitings induces a patient agent to wait. A patient agent observing only a waiting knows that she is in position 2 and by waiting she can induce the last patient agent to wait. Hence, when observing only a waiting, waiting is the best response for a patient agent. Consequently, if a patient agent observes a waiting followed by a withdrawal, then she knows that the agent at position 2 must have been an impatient agent, so by waiting she can induce the last patient agent to wait. Then, for a patient agent observing a waiting or a waiting followed by a withdrawal, the best response is to wait. A patient agent observing nothing knows that she is the first in the line. By

[6] The most simple example is that of three agents with two impatient agents, where both have to wait to make waiting worthwhile. Coordination in that setup is easy and does not give the flavour of the argument we will use in the general case.

waiting she induces the other patient agents to wait according to the previous results, so for a patient agent in position 1 the best response is to wait. As a consequence, if a patient agent observes a withdrawal, then she knows that it must have been an impatient agent. Each patient agent infers this. Then, the best response when observing a withdrawal is to wait, because the subsequent patient agents will know that nobody lied. Hence, the patient agent at position 3 will wait, because this way she induces the last patient agent to wait as well. Thus, waiting is best response for a patient agent when observing

- nothing,
- a withdrawal,
- a waiting,
- (waiting, withdrawal),
- (withdrawal, waiting),
- any history containing two waitings.

Thus, as the game unfolds for a patient agent no information set may emerge to which withdrawal is the best response. Consequently, there will be no runs.

Before going to the general model, let us consider the importance of the order. Note that in the reasoning we used the exact order of moves to get this result. Since the bank observes the exact history, this information is available in the economy. If only aggregate numbers of waitings and withdrawals without order were observable, then we do not get this result.[7] In the Appendix 1, we show how multiple equilibria emerge when only unordered aggregates are observed. Knowing the exact order of previous actions helps to verify the truthfulness of the history.

3.2 The general case

Each agent entering the bank may choose either to wait (wa) or withdraw (wi). Denote by ω_i and μ_i the number of withdrawals and waitings in the history of agent in position i. Let $h_i \in H_i$ be the history observed by agent in position $i \in \{1, 2, ..., n\}$, where H_i is the set of feasible histories. Therefore, it contains all the possible permutations of $\omega_i \in \{0, 1, 2, ...i - 1\}$ withdrawals and $\mu_i \in \{0, 1, 2, ...i - 1\}$ waitings such that $\omega_i + \mu_i = i - 1$. Denote by $\omega \in \{0, 1, 2, ...n\}$ the total number of withdrawals in period 1. The total number of waitings is given by $\mu = n - \omega$.

A pure strategy for agent i is a map $\mathbf{s}_i : \boldsymbol{\theta}_i \times H_i \rightarrow \{wi, wa\}$. Let $\mathbf{s}_i^j = (\mathbf{s}_i, \mathbf{s}_{i+1}, ..., \mathbf{s}_j)$ denote the strategies of agents beginning with agent i up to agent j. Notationally, $\mathbf{s}_i$ denotes the strategy, while s_i will stand for the play implied by $\mathbf{s}_i$. Hence, $h_i = (s_1, s_2, ..., s_{i-1})$.

[7] Smith and Sorensen (1998) shows in more detail the difficulties of this approach.

Since *ex ante* agents ignore their type and position in the line, a strategy is $\mathbf{s} = \mathbf{s}_1 \times \mathbf{s}_2 \times ... \times \mathbf{s}_n$, where $\mathbf{s}_i$ is defined as before for any $i \in [1, n]$. Before the game starts each agent has to specify what to do in any position upon observing any possible history given their type. Being truthful means that patient agents wait, whereas impatient ones withdraw. We say that the first best obtains if all agents act truthfully.

Regarding the formation of beliefs, we will use two restrictions:

1. a waiting at any position reveals that it must have been a patient agent, and

2. if for a patient agent the dominant strategy given history h_i is to wait, then observing a withdrawal reveals that the agent in position i is impatient.

The first restriction eliminates the possibility of impatient agents acting mistakenly, whereas the second one does the same with patient agents. Since impatient agents do not make mistakes, we have $\mathbf{s}_i : imp \times H_i \rightarrow wi$ for all i, so impatient agents always withdraw. Hence, we focus on the truthfulness of patient agents' actions.

These restrictions amount to say that agents are rational and we assume that it is common knowledge. The restrictions also show that beliefs depend on the history. These assumptions allow us to use the iterated elimination of strictly dominated strategies. It makes possible that for a subset of histories agents can predict how later-coming agents will behave if they choose to wait.

3.2.1 Alignment is public knowledge

It is instructive to see what happens if we eliminate the uncertainty of alignment. Suppose that the alignment, that is the type vector of agents is publicly known. This setup allows a patient agent to know how many patient agents have acted before her and how many of them have been truthful. She also knows the continuation alignment and may anticipate the decision of subsequent agents. By eliminating the uncertainty about the alignment we may apply standard backward induction to find the best responses, since every player starts a new subgame. We have the following result.

Proposition 1 *When the alignment is public knowledge, in the unique subgame perfect equilibrium each agent acts truthfully.*

Proof. See Appendix 2. ■

The intuition of this result is the following. The last patient agent's decision is straightforward. If there have been enough waitings before, so that with her waiting the period-2 payment is high enough, then she waits, otherwise she withdraws. Anticipating this decision, the next to the last patient agent's decision is of the same nature, and by moving backwards all patient agents' decision rule becomes clear. Given these rules, as the game unfolds the first best obtains.

3.2.2 Alignment is unknown

When alignments are not observable, agents cannot apply the previous reasoning, because patient agents do not have complete information about the game. This implies that subgame perfection cannot be used in this setup, because histories generally are compatible with many possible alignments and consequently there are several potential continuation alignment. Hence, agent i generally does not start a proper subgame. The nice feature of the model when the alignment is known is that you know exactly what has happened (how many patient agents have lied) and you can predict exactly what will happen (how many later-coming agents will wait). Therefore, patient agents do not need beliefs. This is not true when the alignment is unknown, but still there are histories for which the best response is clear regardless of beliefs. For any patient agent at any position,

$$BR_k(h_k \mid \mu_k \geq \mu_l - 1) = wa, \tag{1}$$

where k is the absolute position (and not the relative one) in the line. It says that if the k^{th} agent's waiting makes waiting a better choice, then a patient agent in this position will wait. This best response can be applied only to a small subset of histories which is not sufficient to determine the equilibria of the game. In the case of histories for which the previous best responses do not apply, beliefs are crucial in finding the optimal action.

The game agents play is one of incomplete information where beliefs are important, so the solution concept we use will be perfect Bayesian equilibrium. Let $F(\theta_{i+1}^n \mid h_i, \theta_i)$ denote agent $i's$ belief about the continuation type vector conditional on the history and $i's$ type.

Definition 1 *The strategy* $\mathbf{s}$ *and the belief* F *is a perfect Bayesian equilibrium if*

$$\sum_{\theta_{i+1}^n} F(\theta_{i+1}^n \mid h_i, \theta_i) u\left[c_1^*, c_2(h_i, s_i, \mathbf{s}_{i+1}^n), \theta_i\right] \geq$$
$$\geq \sum_{\theta_{i+1}^n} F(\theta_{i+1}^n \mid h_i, \theta_i) u\left[c_1^*, c_2(h_i, \tilde{s}_i, \mathbf{s}_{i+1}^n), \theta_i\right]$$

for all i*, and if* $F(\theta_{i+1}^n \mid h_i, \theta_i)$ *is consistent with Bayes' rule whenever possible.*

Notice that the difficulty lies in the fact that h_i, in general, is compatible with several θ_1^{i-1}, because any withdrawal may be due to a misrepresenting patient agent. Given $\mathbf{s}$, using Bayes´ rule $F(\theta_{i+1}^n \mid h_i, \theta_i)$ determines what agent i expects to be the total number of waitings at the end of period 1 which defines her payoff if she decides to wait. As we will show, for the relevant histories the two restrictions we specified before help a lot to determine in a rational way players' type when by observing their actions it cannot be inferred, so Bayes' rule will be of second order.

A special case of $F(\theta_{i+1}^n \mid h_i, \theta_i)$ is when agent i believes that all previous actions have been truthful. We will introduce an even stricter definition for truthful history.

Definition 2 *We call a history truthful, if using restrictions 1 and 2 it can be unambigously verified that all previous actions have been truthful.*

Formally, a truthful history is one where $h_i = \theta_1^{i-1}$.[8] It implies that $F(\theta_{i+1}^n \mid h_i, \theta_i) = F(\theta_{i+1}^n \mid \theta_1^i)$, so there are $p-(\mu_i+\theta_i)$ patient and $n-p-(\omega_i+(1-\theta_i))$ impatient subsequent agents and any continuation alignment is equiprobable. Note that we require that using the restrictions agents be able to verify the truthfulness of the history. By our common knowledge assumption any agent able to verify the truthfulness of a history can be sure that all other agents do the same when observing the same history. When we speak about a truthful history, then it is equivalent to speaking about a degenerate belief where it can be verified that all previous actions have been truthful. For example, when observing the history (wa, wa, wa) any agent should come to the conclusion that it is due to three patient agents. If a patient agent observes a truthful history, then she knows her relative position among the patient agents and she knows that the other patient agents will know it as well.

Our last definition before the main result of this section concerns implementability.

Definition 3 *The first best is strongly implementable if* $\mathbf{s}_i(\theta_i, \theta_1^{i-1}) = \theta_i$ *and* $F(\theta_{i+1}^n \mid \theta_1^i)$ *for all i is the unique perfect Bayesian equilibrium of the game.*

The definition says that if for any agent the belief to observe the truthful history and the strategy to act truthfully is the unique perfect Bayesian equilibrium, then as a consequence the first best obtains

Proposition 2 *The first best is strongly implementable.*

Proof. See Appendix 3. ■

To get the intuition behind the proof, consider the following informal analysis. A patient agent observing $p-1$ waitings at any position knows with certainty that she is the last patient agent, so her optimal action is to wait. Thus, at any equilibrium the strategy for a patient agent when observing $p-1$ waitings and any number of withdrawals[9] should be to wait. Otherwise, she would like to deviate unilaterally, because waiting dominates withdrawal.

Consider now the history consisting of $p-2$ waitings and no withdrawals. All previous agents must have been truthful, so knowing the best response of a patient agent observing $p-1$ waitings, a patient agent´s optimal action is to wait. But then the history $((p-2)\ wa, wi)$ reveals that the last agent must

[8] We have defined strategies as waiting (wa) and withdrawal (wi) and types as patient (pat) and impatient (imp), so when we put $h_i = \theta_1^{i-1}$, then we translate in a straightforward manner wa into pat and wi into imp.

[9] It can be at most $n-p$.

have been an impatient one. Note that this is just an argument involving the elimination of dominated strategies. Therefore, a patient agent observing this history knows that she is the $(p-1)^{th}$ patient agent in the line and her best response is to wait, because this induces the last patient agent to wait as well. We may apply the same line of reasoning to show that for any history beginning with $p-2$ waitings any subsequent withdrawal must be a truthful one. A patient agent upon observing such a history knows exactly her relative position and she knows also what the last patient agent will wait, so her best response is to wait. Hence, at any equilibrium the strategy for a patient agent when observing a history which begins with $p-2$ waitings should be to wait. Otherwise, she would like to deviate unilaterally. On the other hand, whenever a patient agent upon observing $p-2$ waitings knows that she is the $(p-1)^{th}$ patient agent in the line[10], her best response is to wait. This is the case, because we have seen that the last patient agent will wait upon observing $p-1$ waitings.

Consider the history consisting of $p-3$ waitings and no withdrawals. By the previous result a patient agent´s best response when observing this history is to wait. Thus, the history $((p-3)\ wa, wi)$ reveals that the last agent must have been an impatient one. Therefore, a patient agent observing this history knows that she is the $(p-2)^{th}$ patient agent in the line. If she waits, then the resulting history will have $p-2$ waitings and a patient agent would know that she is the $(p-1)^{th}$ patient agent in the line, and her best response would be to wait. The same argument holds for any history beginning with $p-3$ waitings and followed by at most $n-p$ withdrawals. At any equilibrium the strategy for a patient agent when observing a history which begins with $p-3$ waitings should be to wait. Otherwise, she would like to deviate unilaterally. Furthermore, whenever a patient agent upon observing $p-3$ waitings knows that she is the $(p-2)^{th}$ patient agent in the line, her best response is to wait. This is the case, because the following patient agent will observe $p-2$ waitings and will know that she $(p-1)^{th}$ patient agent in the line, so her best response is to wait, as it will be the last patient agent's best response.

Hence, the best response when observing a history which begins with $p-1, p-2, p-3$ waitings is to wait. We can continue along the same lines to show that at any equilibrium the strategy for a patient agent when observing a history which begins with $[0, p-1]$ waitings should be to wait. Notice that what we have shown is that the best response to truthful histories is to be truthful. As the game begins, if the first agent is a patient one, then she will be truthful, because she observes a truthful history. Hence, the second agent can be sure to observe a truthful history as well, implying that she will also act truthfully. The same logic ensures that any later-coming agent can be sure to observe a truthful history to which the best response is to be truthful, so the first best obtains.

The reasoning excludes the possibility of equilibria where patient agents at the beginning of the line withdraw because they believe that later-coming patient agents will withdraw as well. If they wait, then they can induce those

[10] That is, she knows that all patient agents before her have been truthful.

later-coming patient agents to wait as well. In Appendix 4 we show using our four-agent example why runs cannot happen in equilibrium.

3.3 Relating to the literature

An alternative interpretation[11] of the sequentiality is imaging a Diamond-Dybvig model where the agents coordinate on a run. Running means that they form a queue at the door of the bank which will serve them in a sequential manner. Diamond and Dybvig's analysis ends there, while ours begins at this point by posing the question: what is the outcome of the game if any agent can observe the actions of those who are in front of her and if she knows that those coming later will know what she did. If you let people decide in this situation, then our result predicts that the patient agents will not withdraw. In turn, it means that the policies studied by Diamond and Dybvig (suspension of convertibility and deposit insurance) are not necessary to avoid the bad outcome. It is enough that agents are aware of the fact that their decision will be observed by later-coming agents to implement the good equilibrium.

Green and Lin obtained a no-run result with a model with huge withdrawal demand uncertainty, allowing the bank to write very sophisticated contracts, but restricting the agents´ information to have some notion about the position without knowing anything about the others´ decision. In contrast, we work with simple contracts and no demand uncertainty, but allow the bank to share available information with the agents. This trade-off in modeling choice does not change the positive result.

4 Only withdrawals observed

Peck and Shell (2003) assert that it is implausible that agents contact the bank at period 1 to say that they do not want to withdraw. It is more natural to think that only those who want to withdraw will go to the bank. Thus, strategy has to be based on the number of previous withdrawals, that is, the possible strategy profile is of the form $\mathbf{s} = (\mathbf{s}_0, \mathbf{s}_2, ..., \mathbf{s}_{n-1})$ where $\mathbf{s}_i : \{imp, pat\} \times i \rightarrow \{wi, wa\}$ for $i = 0, 1, ..., n-1$ tells what action to take when being either type and observing i withdrawals. To exclude trivial cases, assume that at least two waitings are needed to make waiting a better choice. Let us construct a run strategy profile for patient agents which allows for the maximum number of withdrawals and which is an equilibrium candidate. The most obvious run strategy candidate is the one prescribing to run when observing any number of previous withdrawals. There is only one potential agent who would like to deviate, namely a patient agent who knows that except her there is no more agent to be served. In our setup, this is equivalent to the last agent in the line. Suppose that everybody up to the last agent has withdrawn and the bank still has some funds. Then, in case that this agent is patient, her optimal decision is to wait and consume more in the next period.

[11] This interpretation was suggested to me by Alfonso Rosa García.

Therefore, our proposed run strategy is

$$\mathbf{s}_i = \begin{cases} wi \text{ if } i < n-1, \\ wa \text{ if } i = n-1 \end{cases}.$$

The game is as follows. Nature picks an alignment, players are called to decide (wait or withdraw) sequentially, and each of them observes the number of previous withdrawals. Again, the idea is that the bank knows how many agents have withdrawn and can share this information with the subsequent agents. Waiting is neither observed by the bank, nor by the agents.

Notice that this game does not fulfill the textbook requirements of a game, because it is neither a strategic nor an extensive game. It is not strategic, since agents may observe actions of agents who have acted previously. The game cannot be properly represented in the extensive form, because agents at different positions may have the same information sets. For example, a patient agent at the first position has the same information as a patient agent at the second position who happens to come after a patient agent who has waited.

Proposition 3 *The proposed run strategy with the corresponding beliefs constitutes a Bayesian-Nash equilibrium.*

Proof. See Appendix 5. ■

The intuition behind this result is easy. Since deviations from the run strategy cannot be observed, no patient agent can induce later-coming agents to wait by waiting. When waiting is observable, then being truthful makes possible that later-coming agents have information about what happened before, and then these agents will find it profitable to be truthful as well. In this setup, being truthful is not revealing, agents do not even know their position, so it is not possible that there be enough information to eliminate run as an equilibrium outcome.

5 Reporting is allowed

Up to this point we have shown that if everybody has to report and the history is observable, then bank run in equilibrium does not occur. Nevertheless, by modifying the game so that only withdrawals are observable, runs appear in equilibrium. A way to bridge the gulf between the results is to allow (but not to require) patient agents to report their waiting. It is a new game, since the available actions (withdraw (wi), wait without reporting (wa), wait and report (r))[12] and the possible information sets are different. Since reporting to the bank in period 1 is not related to consumption, we allow for the possibility that it is costly.[13] Intuitively, a patient agent would like to report, because sending

[12] We do not allow to report and withdraw.

[13] How are reporting costs in real life? Our guess is that they are rather small as a consequence of technological advances, like Internet banking. Notice that in Green and Lin (2003) the compulsory reporting is not costly.

this signal could induce subsequent patient agents not to withdraw, and have a high period-2 payment.

Assume a nonnegative and uniform cost for reporting in utility terms and denote it by k. If $k > u(c_2^*) - u(c_1^*)$, then the cost is so high that it does not compensate for the potential gain in utility, so to make reporting a real option suppose the opposite. Otherwise we have the previous setup where run is an equilibrium outcome.

Notice that if we change the setup by adding additional information, then we are back to previously analyzed cases. If both the position and the alignment were known, patient agents would know their relative position. Hence, the game would simplify to that in section 3.2.1 with the same outcome. Patient agents would not report, because it is costly and redundant. If only the position ($i = 1, 2, ...n$) was known, a patient agent would know exactly how many patient agents have waited without reporting. It is simply $(i-1) - (\omega_i + \rho_i)$, that is the difference between all previous actions and all observable actions. It means that both waitings and withdrawals are observable, so we are back to section 3.2.2. with the same outcome. The costly reporting is not needed to obtain the first best, so patient agents would not use it. Nevertheless, in this setup neither the position, nor the alignment is known. To give some flavour of this new game consider the following example.

5.1 Example - Four agents

Suppose that we have the same example as in section 3.1 with an impatient and three patient agents and to make waiting worthwhile.no patient agent should withdraw.

Consider the observable history (r). Clearly, if the history contains also an unobserved waiting, then for a patient agent the best response is to wait without reporting. If there is no unobservable action in the history, then withdrawal is dominated by reporting, because after reporting the last patient agent would observe two reports which would make her wait and the reporting agent would have $u(c_2^*) - k > u(c_1^*)$. Therefore, a patient agent observing a report will not withdraw. As a consequence, when observing (r, wi) agents know that the withdrawal must have been truthful. Hence, for a patient agent observing this history reporting dominates withdrawal, and applying the previous reasoning yields that the best response is to wait without reporting. Knowing this, a patient agent´s best response observing (r) is also to wait without reporting.

Let us see what happens if a patient agent observes (wi). We have seen that when the history begins with a report, then given any of the possible ensuing histories later-coming patient agents will not withdraw.[14] Consequently, for a patient agent who observes nothing reporting dominates withdrawal, so this agent will not withdraw. Therefore, if an observable history begins with a withdrawal, it must have been a truthful one. When observing (wi, r) reporting dominates withdrawal, since when there are two reports in any observable

[14] A patient agent would best respond by withdrawing to an observable history (r, wi, wi), but by our previous argument it cannot arise.

history, then the next patient agent (if there is any) will wait without reporting. Again, since the unique impatient agent has already withdrawn and no patient agent observing (wi, r) withdraws, the best response is to wait without reporting. It implies also that when observing (wi) reporting dominates withdrawal, because the ensuing information sets surely lead to higher payoffs than c_1^*. Moreover, waiting without reporting is the best response, because when observing a withdrawal a patient agent knows that it was done by the impatient agent and if there are any later-coming patient agents, then those agents will not withdraw.

As we have seen, if a patient agent does not observe anything, then she will not withdraw. But, will she report? No, since for a patient agent the best response to the observable history (wi) is to wait without reporting, so the best response to observing nothing is to wait without reporting. Hence, when observing either $(\emptyset)$ or (wi) the best response is to wait without reporting, so as the game unfolds the unique equilibrium which arises is the first best. Notice that in the unique equilibrium patient agents do not report.

5.2 The general case

The information set consists of the history which is observable and the own type. We denote by $H^{obs}_{\omega_j,\rho_j}$ the set of observed histories containing any permutation of $\omega_j \in \{0, 1, 2, ...n - 1\}$ withdrawals and $\rho_j \in \{0, 1, 2, ...p - 1\}$ reports. Denote any generic element of this set by $h^{obs}_{\omega_j,\rho_j}$. The set $(H^{obs}_{\omega_k,\rho_k})_{\substack{\omega_k \geq \omega_j, \rho_k \geq \rho_j \\ \omega_k+\rho_k > \omega_j+\rho_j}}$ represents the possible observable continuation histories. Notice taht it is possible that two (or even more) patient agents observe the same observable history.

Due to the unobservability of waitings, an agent observing any history in $H^{obs}_{\omega_j,\rho_j}$ does not know her position, she just knows that she is at least in position $\omega_j + \rho_j + 1$ and at most in position $\omega_j + p$. The range of possible positions is $p - \rho_j - 1$ which makes the uncertainty larger than in previous setups.

An agent's strategy is based on her type and the observable history. A pure strategy for an agent is a map $\mathbf{s}(\boldsymbol{\theta}, H^{obs}) : \{imp, pat\} \times H^{obs} \to \{wi, wa, r\}$, where $H^{obs} = \times(H^{obs}_{\omega_j,\rho_j})_{\substack{\omega_j \in \{0,1,2,...n-1\} \\ \rho_j \in \{0,1,2,...p-1\}}}$ is the set of all possible observable histories. Therefore, each agent has to specify what to do when observing any possible history and being of either type. We focus on patient agents, because impatient agents always withdraw.

We will modify in a natural way the restrictions on the formation of beliefs:

1. a reporting at any position reveals that it must have been a patient agent, and
2. if for a patient agent given history $h^{obs}_{\omega_j,\rho_j}$ reporting dominates withdrawal, then an observed withdrawal following the history must be due to an impatient agent.

The first restriction is equivalent to saying that impatient agents always withdraw. The second one states that patient agents never play dominated strategies.

Denote by $\theta(\{imp, pat\}, h^{obs}_{\omega_j,\rho_j})$ a possible continuation alignment which depends on the type and the observed history $h^{obs}_{\omega_j,\rho_j}$. Then, $G(\theta(\{imp, pat\}, h^{obs}_{\omega_j,\rho_j}))$ the denotes the distribution of the possible continuation alignment which is the belief patient agents use to determine their action.

Definition 4 *The strategy* $\mathbf{s}$ *and the belief* G *is a perfect Bayesian equilibrium if*

$$\sum_{\theta(\{imp,pat\},h^{obs}_{\omega_j,\rho_j})} G(\theta(\{imp, pat\}, h^{obs}_{\omega_j,\rho_j}))u\left[c_1^*, c_2(h^{obs}_{\omega_j,\rho_j}, s(\boldsymbol{\theta}, h^{obs}_{\omega_j,\rho_j}), s(\boldsymbol{\theta}, (h^{obs}_{\omega_j,\rho_j})_{\substack{\omega_k\geq\omega_j,\rho_k\geq\rho_j \\ \omega_k+\rho_k>\omega_j+\rho_j}}))\right] \geq$$

$$\geq \sum_{\theta(\{imp,pat\},h^{obs}_{\omega_j,\rho_j})} G(\theta(\{imp, pat\}, h^{obs}_{\omega_j,\rho_j}))u\left[c_1^*, c_2(h^{obs}_{\omega_j,\rho_j}, \tilde{s}(\boldsymbol{\theta}, h^{obs}_{\omega_j,\rho_j}), s(\boldsymbol{\theta}, (h^{obs}_{\omega_j,\rho_j})_{\substack{\omega_k\geq\omega_j,\rho_k\geq\rho_j \\ \omega_k+\rho_k>\omega_j+\rho_j}}))\right]$$

for all $h^{obs}_{\omega_j,\rho_j} \in H^{obs}$*, and if* $G(\theta(\{imp, pat\}, h^{obs}_{\omega_j,\rho_j}))$ *is consistent with Bayes' rule whenever possible.*

Notice that $\theta(\{imp, pat\}, h^{obs}_{\omega_j,\rho_j})$ implies what an agent believes that has happened before, and together with the strategy it also tells what the agent believes about later-coming agents' decisions. Hence, a patient agent can calculate the expected utility of either action and she chooses the one yielding the highest expected utility.

We will modify slightly the definition of truthful history.

Definition 5 *We call a history truthful, if using restrictions 1 and 2 it can be unambigously verified that no patient agent has withdrawn.*

As in section 3, truthful history is just another way of formulating a belief which can be verified with the specified restrictions.

Hence, we have a game where agents may wait without reporting, wait and report incurring cost k or withdraw, and where the two last actions are observable. In this game, agents who decide observe the exact order of reports and withdrawals.

Proposition 4 *The first best is strongly implementable.*

Proof. See Appendix. ■

The intuition of the proof is the following. Suppose that a patient agent observes a history with $p-1$ reports. Then, she can be sure to be the last patient agent, and the best she can do is to wait which yields her a sure payment of c_2^*. She does not need to spend on costly reporting. If a patient agent observes a history which starts with $p-2$ reports (followed by no withdrawals), then she knows that no patient agent has withdrawn. For this agent reporting dominates withdrawal, because if there is a later-coming patient agent[15], then she will wait, so the ensuing utility $(u(c_2^*) - k)$ is higher than that of withdrawing. But she can do even better than reporting. Let's consider why is waiting a better choice. Since no patient agent would withdraw upon observing this history, any later-coming patient agent will know that the withdrawal after $p - 2$ reports must be a truthful one. Therefore, a patient agent observing a history which starts with $p-2$ reports and is followed by a withdrawal knows that no patient agent has withdrawn, so for her reporting will dominate withdrawal. Notice that the same argument holds for any history which starts with $p - 2$ reports followed by withdrawals. Given that later-coming patient agents will not withdraw upon observing a history beginning with $p-2$ reports and followed by at most $n-p$ withdrawals, the optimal action for a patient agent when observing any history starting with $p - 2$ reports is to wait and not incur the cost of reporting. Now consider an observed history consisting of $p - 3$ reports. For a patient agent observing this history reporting dominates withdrawal. Hence, the history $p-3$ reports followed by a withdrawal reveals that the withdrawal has been a truthful one. Given this history, reporting again dominates withdrawal for patient agents, because once there are $p - 2$ reports and a truthful withdrawal no subsequent patient agent would withdraw. This dominance argument holds for any history beginning with $p-3$ reports, so the best a patient agent can do when observing such a history is to wait. The same argument holds for histories starting with less and less reports and waiting is a patient agent's optimal action when observing any history starting with 0 reports and followed by at most $n - p$ withdrawals. Note that although agents have the possibility report, they will not do it in the unique equilibrium. Hence, the mere existence of the possibility of reporting is enough to overturn a game with a possible run into one where run does not occur in equilibrium.

The possibility of reporting can be seen as a metaphor of richer communication between the bank and its depositors. While the no-run result by Green and Lin (2003) rests on complex contracts, our no-run result impinges on the possibility of richer communication.

Notice that while the no-run result in section 3 rests upon observing all previous actions, here agents do not observe all previous actions, they generally do not even know with certainty their position.

6 Conclusion

Most of the literature on bank runs uses a simultaneous-move approach to model the depositors´ decision. In contrast, we model it using a sequential focus. We find that in an environment where each previous action is observable, the coordination problem pointed out by Diamond and Dybvig does not emerge. When we restrict the observable information to withdrawals, then runs appear in equilibrium. Nonetheless, they disappear if we allow depositors to report the bank their decision to wait. Besides the no-run result, this last case is interesting

[15] It is possible that there has been before a patient agent who waited.

because in equilibrium no reports are made to the bank. The mere existence of reports is enough to obtain the first best.

Our results rest heavily on the concept of the bank as a benevolent institution which serves the depositors; an assumption adopted by much of the literature. When taking into account that the bank possibly follows self-interest as well (see Andolfatto and Nosal (2008)), then the potential agency problems may question our results, although competition in the banking sector may mitigate these problems.

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8 Appendix 1

Consider the same example, but now the previous actions are unordered. The optimal decision for a patient agent in the last position is as before. When she observes 2 withdrawals and a waiting she withdraws, otherwise she waits. We focus on patient agents at the first three positions. The following table summarizes the clear best responses.

info set	Best Response	reason
(2 w)	w	$u(c_2(\mu \leq 2) < u(c_1^*)$ and $3c_1^* < 4$
(nw,w)	?	
(2 nw)	nw	$u(c_2(\mu = 3) > u(c_1^*)$
(w)	?	
(nw)	nw	the BR to (2 nw) and (2 nw,w) is nw
$\emptyset$	?	

A patient agent at position 3 observing a waiting and a withdrawal knows that if the second agent was patient and observed a waiting, then she surely would have waited, so there are two possibilities to have this information set. Either the second agent observed a withdrawal and waited, or the first agent waited and the second agent was an impatient one. The second case does not involve misrepresentation. In the first case the only possibility of lying happens if the first agent is patient and withdraws. She may do so, because she is not sure how a patient agent observing a waiting and a withdrawal decides. Thus, what a patient agent at the first position does depends on what she believes a patient agent observing a waiting and a withdrawal will do, and viceversa. Therefore, we may have multiple equilibria with patient agents in either of these information sets withdrawing or waiting. Concretely, if we complete the previous table with strategies prescribing withdrawal, then it is an equilibrium for certain parameters and utility functions. If $R = 1,1$ and $\gamma = 6$ for the utility function $u(c) = c^{1-\gamma}/(1-\gamma)$,[16] then given this strategy profile the utility of withdrawing is $-0,149$, while by waiting the expected utility for a patient agent at the first position is $-0,205$, so a patient agent at the first position would withdraw. If a patient agent at the first position withdraws, then a patient agent observing a withdrawal cannot do better by waiting instead of withdrawing. As a consequence, the information set (nw,w) cannot arise. No

[16] For this utility function we have

$$c_1^* = \frac{4}{1+3R^{\frac{1-\gamma}{\gamma}}}, c_2^* = R^{\frac{1}{\gamma}} c_1^*.$$

patient agent will wait, so we have a run equilibrium. By prescribing waiting, the first best obtains. In this setup, agents cannot use the order to check the truthfulness of the actions previously taken.

9 Appendix 2

Proof. The last patient agent´s best response in any known alignment is to wait if and only if with her waiting the number of waitings at the end of period 1 (μ) is so high that $c_2(\mu) > c_1^*$. Denote the minimum μ for which it is true by μ_l,[17] so if the last patient agent observes at least $\mu_l - 1$ waitings, then she will wait, otherwise she withdraws. Note that there is no uncertainty here. Now consider the next to the last patient agent. Knowing what the last patient agent will do, her best response is to wait if she observes at least $\mu_l - 2$ waitings, otherwise she withdraws. By following the same line of argument, it is easy to write in general the best response of any patient agent:

$$BR_\kappa = \begin{cases} wa \text{ if } \mu_\kappa \geq \mu_l - (p+1-\kappa) \\ wi \text{ otherwise} \end{cases},$$

for $\kappa \in [1,p]$ where the subscript κ denotes the κ^{th} patient agent in the line.

Now consider how the game unfolds. The first patient agent waits, because $0 \geq \mu_l - p$. The second patient agent also waits, because $1 \geq \mu_l + 1 - p$, and so on. In the end, all patient agents will wait yielding $\mu = p$, so the first best obtains. Note that to get this result we need less than knowing with certainty the alignment. It is enough that patient agents know their position among the patient agents. ■

10 Appendix 3

The proof has two parts. First, we show that the best response when observing a truthful history is to be truthful. In the second step, we argue that as the game unfolds, these best responses lead to the first best.

Denote by $H^{tr}(\hat{\mu})$ the set of truthful histories which contain $\hat{\mu}$ waitings (and any $\hat{\omega} \in [0, n-p]$ withdrawals). Notice that characterizing features of the set are the number of waitings and truthfulness, but not the position of the agent observing any element of the set. This is in line with our previous finding that not the absolute position, but the relative position among the patient agent is what really matters for a patient agent.

Lemma 1 *Assume that once an element in $H^{tr}(\hat{\mu})$ is reached all subsequent agents will act truthfully, that is, $s_i(\theta_i, h^{tr}(\mu_i \geq \hat{\mu})) = \theta_i$ for $i = \hat{\mu} + \hat{\omega} + 1, ..., n$, where $h^{tr}(\mu_i \geq \hat{\mu}) \in H^{tr}(\mu_i \geq \hat{\mu})$. Then, for the set of truthful histories which contain $\hat{\mu} - 1$ waitings (and any $\hat{\omega} \in [0, n-p]$ withdrawals), we have $s_{\hat{\mu}+\hat{\omega}}(\theta_{\hat{\mu}+\hat{\omega}}, h^{tr}(\hat{\mu} - 1)) = \theta_{\hat{\mu}+\hat{\omega}}$.*

[17] Thus, it is given by $c_2(\mu_l) > c_1^* \geq c_2(\mu_l - 1)$, and notice that $\mu_l \leq p$.

Proof. The lemma assumes that once a truthful history containing $\hat{\mu}$ waitings and at most $n-p$ withdrawals is reached, for any possible continuation alignment later-coming patient agents will wait. Therefore, the only equilibrium strategy when observing a truthful history with $\hat{\mu}-1$ waitings is to act truthfully.

It is easy to see that if a patient agent observes $h^{tr}(\hat{\mu}-1) \in H^{tr}(\hat{\mu}-1)$, then by waiting she will cause a history which belongs to $H^{tr}(\hat{\mu})$. By our assumption, all subsequent agents will be truthful, so the first best obtains yielding the highest obtainable payoff to the patient agents. Since any truthful history is equivalent to a degenerate belief, given such a history the unique perfect Bayesian equilibrium strategy is to be truthful, since there is no unilateral profitable deviation. ■

Notice that the previous induction step can be used repeatedly.

Corollary 1 *Assume that for the set of truthful histories which contain $\hat{\mu}$ waitings (and any $\hat{\omega} \in [0, n-p]$ withdrawals) we have $s_i(\theta_i, h^{tr}(\mu_i \geq \hat{\mu}) = \theta_i$ for $i = \hat{\mu} + \hat{\omega} + 1, ..., n$. Then, for the set of truthful histories which contain $\acute{\mu} \in [0, \hat{\mu}-1]$ waitings (and any $\hat{\omega} \in [0, n-p]$ withdrawals), we have $s_{\acute{\mu}+\hat{\omega}+1}(\theta_{\acute{\mu}+\hat{\omega}+1}, .) = \theta_{\acute{\mu}+\hat{\omega}+1}$.*

Proof. In the previous lemma we have shown the case when $\acute{\mu} = \hat{\mu} - 1$. What happens if a patient agent observes a truthful history with $\acute{\mu} = \hat{\mu} - 2$ and $\hat{\omega} \in [0, n-p]$ withdrawals? By waiting, the resulting history will be a truthful one with $\hat{\mu}-1$ waitings and $\hat{\omega} \in [0, n-p]$ withdrawals. By our common knowledge assumption all subsequent agents will know that up to agent $\hat{\mu} + \hat{\omega} - 1$ all actions have been truthful, so by the previous lemma all subsequent actions will be truthful as well. The resulting first best yields the highest possible payoff to any patient agent, so there is no unilateral profitable deviation. Hence, given the belief embodied in the history the unique perfect Bayesian equilibrium strategy is to be truthful. The same argument can be applied to any truthful history with less and less waitings. ■

Consider a patient agent who observes a history which contains $p-1$ waitings and $\omega^i \in [0, n-p]$ withdrawals, so each patient agent other than the one who observes the history has waited. Any such history is a truthful one, and the only equilibrium strategy is $s_i(pat, h^{tr}(p-1)) = wa$ because it leads to the first best which yields the highest obtainable payoff. Therefore, we may apply the corollary to this set of truthful histories.

Lemma 2 *For the set of truthful histories which contain $\acute{\mu} \in [0, p-1]$ waitings (and any $\hat{\omega} \in [0, n-p]$ withdrawals), we have $s_{\acute{\mu}+\hat{\omega}+1}(\theta_{\acute{\mu}+\hat{\omega}+1}, .) = \theta_{\acute{\mu}+\hat{\omega}+1}$.*

Proof. Apply corollary to $H^{tr}(p-1)$. ■

Proposition 5 *The strategy $\mathbf{s}_i(\boldsymbol{\theta}_i, \theta_1^{i-1}) = \theta_i$ and the belief $F(\theta_{i+1}^n \mid \theta_1^i)$ for all i is the unique perfect Bayesian equilibrium of the game.*

Proof. Consider the history consisting of $\acute{\mu}$ waitings and no withdrawal, where $\acute{\mu} \in [0, p-1]$. The unique compatible belief is that it is a truthful history, so by the previous lemma $\mathbf{s}_{\acute{\mu}+1}(\theta_{\acute{\mu}+1}, \theta_1^{\acute{\mu}}) = \theta_{\acute{\mu}+1}$. As a consequence, the history starting with $\acute{\mu}$ waitings and followed by a withdrawal reveals that the last agent must have been an impatient agent. Therefore, a patient agent observing this history knows that it is a truthful one, so $\mathbf{s}_{\acute{\mu}+2}(\theta_{\acute{\mu}+2}, \theta_1^{\acute{\mu}+1}) = \theta_{\acute{\mu}+2}$. This argument shows that any history starting with $\acute{\mu} \in [0, p-1]$ waitings must be a truthful one, so the previous lemma applies to them. Now consider how the game unfolds. If the first agent is patient, then her belief is $F(\theta_2^n \mid \emptyset, pat) = F(\theta_2^n \mid pat)$ which corresponds to our definition of a truthful history. The previous lemma ensures that her optimal action is to wait. Thus, the second agent can be sure to observe a truthful history, so her optimal action is to act truthfully as is the case for each later-coming agent. Agents at any position can be sure to observe a truthful history to which the unique equilibrium strategy is to be truthful. ■

As a consequence of the proposition we have the following corollary.

Corollary 2 *The first best is strongly implementable.*

11 Appendix 4

Intuitively, we may think that a run is possible, if - for instance - a patient agent at the first position believes that the strategy of any later-coming patient agent is to withdraw, so it seems optimal for her to withdraw as well. We will show why this argument does not hold using the four-agent example presented at the beginning of section 3.

To remind, there are 3 patient and an impatient agent. Suppose that $u(c_2(\mu = 3)) > u(c_1^*), u(c_2(\mu \leq 2) < u(c_1^*)$ and $3c_1^* < 4$, so patient agents would only want to wait if all the other patient agents wait. We focus on the decisions of patient agents. The optimal decision for a patient agent in the last position is easily defined. When she observes a history with 2 withdrawals she withdraws, otherwise she waits. We know also that if a patient agent observes any history containing 2 waitings, then her best response is to wait. Thus, in any equilibrium we should have

$$s_3(wa, wa) = wa,$$
$$s_4(wa, wa, wi) = s_4(wa, wi, wa) = s_4(wi, wa, wa) = wa,$$
$$s_4(wa, wi, wi) = s_4(wi, wi, wa) = s_4(wi, wa, wi) = wi.$$

Run happens if at least one of the patient agents withdraws. When patient agents at the beginning of the line wait, then later-coming patient agents will wait as well, so to generate a run we should have the first patient agents withdraw. Hence, we propose the following strategies:

$$\begin{aligned} s_1(\emptyset) &= wi, \\ s_2(wi) &= s_2(wa) = wi, \\ s_3(wa, wi) &= s_3(wi, wa) = s_3(2wi) = wi. \end{aligned}$$

If these strategies really form an equilibrium, then agents at any position should observe histories consisting only of withdrawals. It is straightforward to compute the beliefs on the proposed equilibrium path, which are the following

$$F(\theta_2^4 \quad | \quad \emptyset, pat) = \begin{cases} pat, pat, imp \text{ with prob. } \frac{1}{3} \\ pat, imp, pat \text{ with prob. } \frac{1}{3} \\ imp, pat, pat \text{ with prob. } \frac{1}{3} \end{cases},$$

$$F(\theta_3^4 \quad | \quad wi, pat) = \begin{cases} pat, pat \text{ with prob. } \frac{1}{3} \\ imp, pat \text{ with prob. } \frac{1}{3} \\ pat, imp \text{ with prob. } \frac{1}{3} \end{cases},$$

$$F(\theta_4^4 \quad | \quad 2wi, pat) = \begin{cases} imp \text{ with prob. } \frac{1}{3} \\ pat \text{ with prob. } \frac{2}{3} \end{cases}.$$

What about the out-of-equilibrium beliefs? These are those which contain waitings. There are two ways of interpreting a waiting. Either it has been a mistake or a strategic deviation. We have supposed that agents do not make mistakes, so waiting must be due to a patient agent. Consequently, we have the following beliefs

$$F(\theta_3^4 \quad | \quad wa, pat) = \begin{cases} imp, pat \text{ with prob. } \frac{1}{2} \\ pat, imp \text{ with prob. } \frac{1}{2} \end{cases},$$

$$F(\theta_4^4 \quad | \quad (wa, wa), pat) = imp \text{ with prob. } 1.$$

Note that we have not specified the beliefs when observing (wa, wi) and (wi, wa). But the strategies and beliefs we have specified up to this point are enough to answer the question whether we have a perfect Bayesian equilibrium or not. Consider a patient agent at the first position. If we forget for a moment the beliefs, then for her there is no profitable unilateral deviation. It is the case, because if the other agents stick to their strategies, then she will have $u(c_2(\mu \leq 2)$ which is less than $u(c_1^*)$. But note, that by waiting she influences the second agent´s belief. This second agent will exclude the possibility that the first agent has been untruthful. Hence, the probability of observing an untruthful first agent decreases from $\frac{2}{3}$ to 0. Consequently, if this second agent happens to be a patient one, then it is optimal for her to deviate from $s_2(wa) = wi$ to $s_2(wa) = wa$, because the last patient agent will wait upon observing two waitings. Anticipating it, a patient agent at the first position will wait, because if the second agent is patient, then she will wait, as will do the last patient agent and the first best obtains. Or, if the second agent is impatient and withdraws, then the patient agent at position 3 will infer that the withdrawal must have been due to impatience, so by waiting she can induce the last patient agent to wait. Thus, we have that a patient agent at the first position would deviate, because she anticipates that later-coming patient agents will deviate as well by responding optimally to her deviation. Given that a patient agent at the first position would like to deviate, a history consisting of a withdrawal reveals that it has been an impatient agent.

Therefore, the proposed strategy is not part of a perfect Bayesian equilibrium. Notice that it does not help if we change $s_2(wa) = wi$ to $s_2(wa) = wa$, because then even without considering the beliefs a patient agent at the first position would deviate from $s_1(\emptyset) = wi$ to $s_1(\emptyset) = wa$.

12 Appendix 5

Proof. Are there any profitable deviations? We assumed that at least two patient agents should wait to make waiting a better choice. Suppose that a patient agent decides to wait. Since it will not be observed, subsequent agents will not know about the deviation, so they have no reason to believe that previous agents have not followed the proposed strategy. Hence, a patient agent cannot induce other patient agents to wait so deviations are not profitable. ■

13 Appendix 6

The idea of the proof is to show that for any arising information set as the game unfolds reporting dominates withdrawal. This dominance reveals that withdrawals following the information sets in question are truthful. Using the dominance argument we can show also that patient agents will not report.

Denote by $H^{tr}_{obs}(\hat{\rho})$ the set of observed truthful histories which contain $\hat{\rho}$ reports and any $\hat{\omega} \in [0, n-p]$ withdrawals. An element of the set is denoted by $h^{tr}_{obs}(\hat{\rho})$.

Lemma 3 *Suppose that once any element of $H^{tr}_{obs}(\hat{\rho})$ is reached, all subsequent patient agents will wait. When observing any element of $H^{tr}_{obs}(\hat{\rho}-1)$, a patient agent's unique equilibrium strategy is to wait.*

Proof. For a patient agent observing an element of $H^{tr}_{obs}(\hat{\rho}-1)$ reporting dominates withdrawal, because, as a consequence, at the end of period 1 $\rho+\mu = p$ which yields a payment higher than c_1^* even to the reporting agents. Therefore, any subsequent withdrawal must be a truthful one, so the emerging continuation histories will be truthful. But then a patient agent does not need to report, so her optimal action given any element of $H^{tr}_{obs}(\hat{\rho}-1)$ is to wait which yields her c_2^*. It is the unique equilibrium strategy, because both withdrawal and reporting would yield a smaller payoff. ■

Corollary 3 *Suppose that once any element of $H^{tr}_{obs}(\hat{\rho})$ is reached, all subsequent patient agents will wait. When observing a history which begins with $\hat{\rho}-1$ reports, the unique equilibrium strategy for a patient agent is to wait.*

Proof. If a history begins with $\hat{\rho}-1$ reports followed by no withdrawal, then the unique belief any agent may have is that it is a truthful history. Consequently, for a patient agent reporting dominates withdrawal, so if the $\hat{\rho}-1$ reports are followed by a withdrawal, then it must be a truthful one. Hence, this history is truthful and again for a patient agent reporting dominates withdrawal. By the same dominance argument, all subsequent withdrawals must be truthful, so any history begining with $\hat{\rho}-1$ reports is a truthful history. Since in neither of these cases would patient agents withdraw, it is not necessary to report, so the optimal action is to wait. ■

Lemma 4 *Suppose that once any element of $H^{tr}_{obs}(\hat{\rho})$ is reached, all subsequent patient agents will wait. When observing any element of $H^{tr}_{obs}(\bar{\rho} \leq \hat{\rho}-1)$, a patient agent's unique equilibrium strategy is to wait.*

Proof. Apply lemma 3 repeatedly. ■

Lemma 5 *For any $h^{tr}_{obs}(\bar{\rho}) \in H^{tr}_{obs}(\bar{\rho})$ such that $\bar{\rho} \leq p-1$ a patient agent's unique equilibrium strategy is to wait.*

Proof. Consider a patient agent who observes any history with $p-1$ reports and at most $n-p$ withdrawals. Her optimal action in any of the cases is to wait. Thus, once any element of $H(p-1)$ is reached, all subsequent patient agents will wait. Therefore, we may apply lemma 4 which completes the proof. ■

Proposition 6 *The strategy*

$$s(\{imp, pat\}, H^{obs}) = \left\{ \begin{array}{c} wa \text{ if patient} \\ wi \text{ if impatient} \end{array} \right.,$$

and the belief for patient agents

$$G(\theta(\{imp, pat\}, h^{obs}_{\omega_j,\rho_j})) = \frac{1}{\binom{n-\omega_j-(k+1)}{p-(k+1)}} \text{ for any } k \in (0, ..., p-1),$$

where k is the hypothesized number of patient agents who have already waited is the unique perfect Bayesian equilibrium of the game.

Proof. The patient agents' beliefs only reflect that given the strategy all possible continuation alignments are equiprobable.

As the game starts, as long as there is no withdrawal any patient agent can be sure that the history is truthful, so by lemma 5 a patient agent's unique equilibrium strategy is to wait. Consequently, as the first withdrawal appears, each agent knows that it must have been truthful, so the history remains truthful, and any patient agent´s optimal action observing it is to wait. By the same line of reasoning, the truthfulness of any later withdrawal can be verified, so a patient agent´s unique equilibrium strategy is to wait. There will be only observed histories (a growing number of withdrawals) for which patient agents will wait, so the unique equilibrium is the first best implying that patient agents will not report. ■

A direct consequence of the proposition is the following corollary.

Corollary 4 *The first best is strongly implementable.*